

Charged Dilation Black Holes as Particle Accelerators

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Abstract

We examine the possibility of arbitrarily high energy in the Center-of-mass(CM) frame of colliding neutral particles in the vicinity of the horizon of a charged dilation black hole(BH). We show that it is possible to achieve the infinite energy in the background of the dilation black hole without fine-tuning of the angular momentum parameter. It is found that the CM energy (\mathcal{E}_{cm}) of collisions of particles near the infinite red-shift surface of the extreme dilation BHs are arbitrarily large while the non-extreme charged dilation BHs have the finite energy. We have also compared the \mathcal{E}_{cm} at the horizon with the ISCO(Innermost Stable Circular Orbit) and MBCO (Marginally Bound Circular Orbit) for extremal Reissner-Nordström(RN) BH and Schwarzschild BH. We find that for extreme RN BH the inequality becomes $\mathcal{E}_{cm}|_{r_+} > \mathcal{E}_{cm}|_{r_{mb}} > \mathcal{E}_{cm}|_{r_{ISCO}}$ i.e. $\mathcal{E}_{cm}|_{r_+=M} : \mathcal{E}_{cm}|_{r_{mb}=(\frac{3+\sqrt{5}}{2})M} : \mathcal{E}_{cm}|_{r_{ISCO}=4M} = \infty : 3.23 : 2.6$. While for Schwarzschild BH the ratio of CM energy is $\mathcal{E}_{cm}|_{r_+=2M} : \mathcal{E}_{cm}|_{r_{mb}=4M} : \mathcal{E}_{cm}|_{r_{ISCO}=6M} = \sqrt{5} : \sqrt{2} : \frac{\sqrt{13}}{3}$. Also for Gibbons-Maeda-Garfinkle-Horowitz-Strominger (GMGHS) BHs, the ratio is being $\mathcal{E}_{cm}|_{r_+=2M} : \mathcal{E}_{cm}|_{r_{mb}=2M} : \mathcal{E}_{cm}|_{r_{ISCO}=2M} = \infty : \infty : \infty$.

1 Introduction

In 2009, Bañados, Silk and West[1] (hereafter BSW) proposed an interesting mechanism that is when two massive dark matter particles falling from rest into the infinite red-shift surface of a extreme spinning BH described by the Kerr metric can collides with an infinite high center-of-mass(CM) energy. What we will now call the BSW effect. Hence BHs may act as a natural Planck scale particle accelerators. Whereas in the case of non spinning BH described by the Schwarzschild metric, the CM energy for colliding particles are finite[2]. In a subsequent paper [3], the authors attempted to compute the flux reaching a distant observer from the annihilation of weakly interacting massive particles in the vicinity of an intermediate mass BH.

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Soon after the appearance of this mechanism in the literature, several authors were criticized this effect from the different perspectives. Particularly in [4] and [5], the authors showed that to operate the BSW process it is indeed require precisely a extremal BH and extremal horizon respectively. Any deviation from extremality, the CM energy is also reduced to the order of $10m_0$ or something less, where m_0 is the mass of the colliding particles. Also in [4] the same authors showed that there is an astrophysical limitations i.e. maximal spin, back reaction effect and gravitational radiation etc. on that CM energy due to the Thorn's limit [6] i.e. $\frac{a}{M} = 0.998$ (M is the mass and a is the spin of the BH). In [7], the authors suggested that using collisional Penrose process the emitted massive particles can only be gain ~ 30 percentage of the initial rest energy of the in-falling particles.

Also, Lake [8] calculated the CM energy at the Cauchy horizon of a static Reissner-Nordström (RN) BH and Kerr BH which is limited. Grib and Pablov [9] investigated the CM energy using the multiple scattering mechanism. Also in [10], the same authors computed the CM energy of particle collisions in the ergo-sphere of the Kerr BH. The collision in the ISCO particles was investigated by the [11] for Kerr BH. Liu et al. [12] studied the BSW effect for Kerr-Taub-NUT(Newman, Unti, Tamburino) space-time and proved that the CM energy depends upon both the Kerr parameter (a) and the NUT parameter (n) of the space-time. In [13], the authors proved that the Kerr-AdS BH space-time could act as a particle accelerator. Studies were done by [14] for RN-de-Sitter BH and found that infinite energy in the CM frame near the cosmological horizon. The authors in [15] studied the particle accelerations and collisions in the back ground of a cylindrical BHs. Studies of BSW effect were performed in [16] for the naked singularity case of different space-times.

In [17], the authors discussed the CM energy for the Kerr-Newman BH. For Kerr-Sen BH, the CM energy is diverging also discussed in [18]. It was discussed in [19] regarding the BSW process for spherically symmetric RN BH. The authors in [20] showed that general stationary charged BHs as particle accelerators. In [21], the author demonstrated that a weakly magnetized BH may behave as a particle accelerators. McWilliams [22] showed that the BHs are neither particle accelerators nor dark matter probes. The author in [23] also showed that the CM energy in the context of the near horizon geometry of the extremal Kerr BHs and proved that the CM energy is finite for any value of the particle parameters. Tursunov et al. [24] have studied the particle accelerations and collisions in case of black string. Fernando [25] has studied the possibility of high CM energy of two particles colliding near the infinite red-shift surface of a charged BH in string theory.

The aim of this present work is to show that an analogous effect of particle collision with a high CM energy is also possible when a BH is described by the *extremal charged dilation space-time*. We choose various collision points say ISCO, MBCO etc. and at that point we compute the CM energy and prove that at the horizon the \mathcal{E}_{cm} is maximum than the MBCO and ISCO for extremal RN BH. We find that for extreme RN BH the ratio becomes $\mathcal{E}_{cm} |_{r_+=M} : \mathcal{E}_{cm} |_{r_{mb}=\left(\frac{3+\sqrt{5}}{2}\right)M} : \mathcal{E}_{cm} |_{r_{ISCO}=4M} = \infty : 3.23 : 2.6$. Which implies

that $\mathcal{E}_{cm}|_{r_+} > \mathcal{E}_{cm}|_{r_{mb}} > \mathcal{E}_{cm}|_{r_{ISCO}}$. For our completeness we also compute the \mathcal{E}_{cm} for Schwarzschild BH and we find the ratio as $\mathcal{E}_{cm}|_{r_+=2M} : \mathcal{E}_{cm}|_{r_{mb}=4M} : \mathcal{E}_{cm}|_{r_{ISCO}=6M} = \sqrt{5} : \sqrt{2} : \frac{\sqrt{13}}{3}$. Finally we show the ratio for GMGHS BH and the ratio becomes $\mathcal{E}_{cm}|_{r_+=2M} : \mathcal{E}_{cm}|_{r_{mb}=2M} : \mathcal{E}_{cm}|_{r_{ISCO}=2M} = \infty : \infty : \infty$. Which is quite different from extreme RN BH and Schwarzschild BH.

The paper is organized as follows. In section 2, we shall study the basic properties of charged dilation BH and in the subsection we shall describe the complete geodesic structure of it. Section 3 will devoted to study the CM energy of the dilation BH and we shall prove that the diverging energy can be obtained from the extreme dilation BH. In section 4, we shall discuss the CM energy of particle collision near the ISCO of an extremal RN BH. We also discuss the CM energy of particle collision near the ISCO of a Schwarzschild BH in section 5. In section 6, we shall describe the CM energy of particle collision near the horizon of the Gibbons-Maeda-Garfinkle-Horowitz-Strominger (GMGHS) BH [26, 27]. Finally in section 7, we have given the conclusions and outlook.

2 Charged dilation metric and its properties:

The metric of a static, spherically symmetric charged dilation BH [27] can be written in Schwarzschild like coordinates:

$$ds^2 = -\mathcal{F}(r)dt^2 + \frac{dr^2}{\mathcal{F}(r)} + \mathcal{R}^2(d\theta^2 + \sin^2\theta d\phi^2) . \quad (1)$$

where the function $\mathcal{F}(r)$ is defined by

$$\mathcal{F}(r) = \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}} . \quad (2)$$

and

$$\mathcal{R}^2(r) = r^2 \left(1 - \frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}} . \quad (3)$$

The associated classical dilation field and gauge potential are

$$e^{2a\phi} = \mathcal{R}^2(r) = r^2 \left(1 - \frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}} , \quad (4)$$

$$\mathcal{A} = \frac{Q}{r}dt, \quad (5)$$

$$F = d\mathcal{A} = -\frac{Q}{r^2}dt \wedge dr. \quad (6)$$

In these equations, r_+ and r_- are constants, which are related to the mass and charge of the BH:

$$M = \frac{r_+}{2} + \left(\frac{1-a^2}{1+a^2}\right) \frac{r_-}{2} \text{ and } Q = \sqrt{\frac{r_+r_-}{1+a^2}} \quad (7)$$

where M is defined as the BH mass and Q is the electric charge of the BH. It may be noted that Q and a are positive. The horizons of the BH are determined by the function $\mathcal{F}(r) = 0$ which yields

$$r_+ = M + \sqrt{M^2 - \left(\frac{2n}{1+n}\right) Q^2} . \quad (8)$$

$$r_- = \frac{1}{n} \left[M + \sqrt{M^2 - \left(\frac{2n}{1+n}\right) Q^2} \right] . \quad (9)$$

where n is defined by

$$n = \frac{1 - a^2}{1 + a^2} . \quad (10)$$

Here r_+ and r_- are called event horizon (\mathcal{H}^+) or outer horizon and Cauchy horizon (\mathcal{H}^-) or inner horizon respectively. $r_+ = r_-$ or $M^2 = \left(\frac{1+n}{2}\right) Q^2$ corresponds to the extreme charged dilation BH.

Case I: When $a = 0$ or $n = 1$, the metric corresponds to RN BH.

Case II: When $a = 1$ or $n = 0$, the metric corresponds to GMGHS BH.

The surface gravity of the dilation BH for both the horizons (\mathcal{H}^\pm) are

$$\kappa_+ = \frac{1}{2r_+} \left(\frac{r_+ - r_-}{r_+} \right)^n \text{ and } \kappa_- = 0 . \quad (11)$$

The BH temperature or Hawking temperature of \mathcal{H}^\pm are

$$T_+ = \frac{\kappa_+}{2\pi} \quad (12)$$

$$= \frac{1}{4\pi r_+} \left(\frac{r_+ - r_-}{r_+} \right)^n \quad (13)$$

and

$$T_- = \frac{\kappa_-}{2\pi} = 0 \quad (14)$$

Then the area of both the horizons (\mathcal{H}^\pm) are

$$\mathcal{A}_+ = 4\pi \mathcal{R}_+^2 = 4\pi r_+^2 \left(\frac{r_+ - r_-}{r_+} \right)^{1-n} \text{ and } \mathcal{A}_- = 4\pi \mathcal{R}_-^2 = 0 . \quad (15)$$

Interestingly, the area of both the horizons go to zero at the extremal limit ($r_+ = r_-$). This feature is quite different from the well known RN and Schwarzschild BH. The other characteristics of this space-time is that there is a curvature singularity at $r = r_-$.

Finally, the entropy of both the horizons (\mathcal{H}^\pm) of the BH reads as

$$\mathcal{S}_+ = \frac{\mathcal{A}_+}{4} = \pi r_+^2 \left(\frac{r_+ - r_-}{r_+} \right)^{1-n} \text{ and } \mathcal{S}_- = \frac{\mathcal{A}_-}{4} = 0 . \quad (16)$$

2.1 Equatorial circular orbit in the charged dilation BH:

This section is devoted to study the properties of the circular geodesics for charged dilation BH in the $\theta = \frac{\pi}{2}$ plane and also compute the ISCO equation for this BH.

To compute the geodesic motion of a test particle in the equatorial plane we set $u^\theta = \dot{\theta} = 0$ and $\theta = \text{constant} = \frac{\pi}{2}$ and follow the pioneer book of S. Chandrasekhar[31]. Thus we may write the Lagrangian density in terms of the metric is given by

$$2\mathcal{L} = -\mathcal{F}(r) (u^t)^2 + (\mathcal{F}(r))^{-1} (u^r)^2 + \mathcal{R}^2(r) (u^\phi)^2 . \quad (17)$$

The generalized momenta reads

$$p_t \equiv \frac{\partial \mathcal{L}}{\partial \dot{t}} = -\mathcal{F}(r) u^t . \quad (18)$$

$$p_r \equiv \frac{\partial \mathcal{L}}{\partial \dot{r}} = (\mathcal{F}(r))^{-1} u^r . \quad (19)$$

$$p_\theta \equiv \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \mathcal{R}^2(r) u^\theta = 0 . \quad (20)$$

$$p_\phi \equiv \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \mathcal{R}^2(r) u^\phi . \quad (21)$$

Here superior dots denote differentiation with respect to affine parameter which is the proper time (τ) for time-like case and for null it is affine parameter(λ).

Since the Lagrangian density does not depend explicitly on the variables ' t ' and ' ϕ ', so p_t and p_ϕ are conserved quantities. Thus one gets,

$$p_t = -\mathcal{F}(r) u^t = -\mathcal{E} = \text{constant} . \quad (22)$$

$$p_\phi = \mathcal{R}^2(r) u^\phi = L = \text{constant} . \quad (23)$$

Solving (22) and (23) for u^t and u^ϕ we obtain

$$u^t = \frac{\mathcal{E}}{\mathcal{F}(r)} . \quad (24)$$

$$u^\phi = \frac{L}{\mathcal{R}^2(r)} . \quad (25)$$

where \mathcal{E} and L are the energy per unit mass and angular momentum per unit mass of the test particle.

Therefore the required Hamiltonian reads as

$$\mathcal{H} = p_t u^t + p_\phi u^\phi + p_r u^r - \mathcal{L} . \quad (26)$$

In terms of the metric the Hamiltonian may be written as

$$\mathcal{H} = -\mathcal{F}(r) (u^t)^2 + (\mathcal{F}(r))^{-1} (u^r)^2 + \mathcal{R}^2(r) (u^\phi)^2 - \mathcal{L} . \quad (27)$$

Since the Hamiltonian is independent of ‘ t' ’, therefore we can write it as

$$2\mathcal{H} = -\mathcal{F}(r)(u^t)^2 + (\mathcal{F}(r))^{-1}(u^r)^2 + \mathcal{R}^2(r)(u^\phi)^2 . \quad (28)$$

$$= -\mathcal{E} u^t + L u^\phi + \frac{1}{\mathcal{F}(r)} (u^r)^2 = \epsilon = \text{const} . \quad (29)$$

Here $\epsilon = -1$ for time-like geodesics, $\epsilon = 0$ for light-like geodesics and $\epsilon = +1$ for space-like geodesics.

Substituting the equations, (24) and (25) in (29), we find the radial equation for charged dilation BH:

$$(u^r)^2 = \dot{r}^2 = \mathcal{E}^2 - \mathcal{V}_{eff} = \mathcal{E}^2 - \left(\frac{L^2}{\mathcal{R}^2(r)} - \epsilon \right) \mathcal{F}(r) . \quad (30)$$

where, the standard effective potential for charged dilation space-time becomes

$$\mathcal{V}_{eff} = \left(\frac{L^2}{\mathcal{R}^2(r)} - \epsilon \right) \mathcal{F}(r) . \quad (31)$$

2.2 Time-like Case:

For time-like case the effective potential is found to be

$$\mathcal{V}_{eff} = \left(1 - \frac{r_+}{r} \right) \left(1 - \frac{r_-}{r} \right)^n \left[1 + \frac{L^2}{r^2} \left(1 - \frac{r_-}{r} \right)^{n-1} \right] . \quad (32)$$

In the extremal case, the effective potential for charged dilation BH reduced to the following form:

$$\mathcal{V}_{eff} = \left(1 - \frac{r_+}{r} \right)^{n+1} + \frac{L^2}{r^2} \left(1 - \frac{r_+}{r} \right)^{2n} . \quad (33)$$

For circular geodesic motion of the test particle of constant $r = r_0$, we must have

$$\dot{r} = 0 \text{ or } \mathcal{V}_{eff} = \mathcal{E}^2 . \quad (34)$$

and

$$\frac{d\mathcal{V}_{eff}}{dr} = 0 . \quad (35)$$

Thus one can obtain the conserved energy and angular momentum per unit mass of the test particle along the circular orbits are:

$$\mathcal{E}_0^2 = \frac{r_0 \left(1 - \frac{r_+}{r_0} \right)^2 \left(1 - \frac{r_-}{r_0} \right)^n \left[2r_0 \left(1 - \frac{r_-}{r_0} \right) + r_- (1 - n) \right]}{[2r_0^2 - (3r_+ + (2n + 1)r_-)r_0 + 2(1 + n)r_+r_-]} . \quad (36)$$

and

$$L_0^2 = \frac{r_0^3 \left(1 - \frac{r_-}{r_0}\right)^{1-n} \left[r_+ \left(1 - \frac{r_-}{r_0}\right) + nr_- \left(1 - \frac{r_+}{r_0}\right)\right]}{[2r_0^2 - (3r_+ + (2n+1)r_-)r_0 + 2(1+n)r_+r_-]} . \quad (37)$$

Case I: When $n = 1$, we retrieve the value of energy and angular momentum of RN BH.

Case II: When $n = 0$, we retrieve the value of energy and angular momentum of GMGHS BH.

Also in the extremal limit, the energy and angular momentum corresponds to

$$\mathcal{E}_0^2 = \frac{r_0 \left(1 - \frac{r_+}{r_0}\right)^{n+2} [2r_0 - (1+n)r_+]}{2[r_0^2 - (n+2)r_+r_0 + (1+n)r_+^2]} . \quad (38)$$

and

$$L_0^2 = \frac{(n+1)r_+r_0^3 \left(1 - \frac{r_+}{r_0}\right)^{2-n}}{2[r_0^2 - (n+2)r_+r_0 + (1+n)r_+^2]} . \quad (39)$$

Case I: When $n = 1$, we retrieve the value of energy and angular momentum of extreme RN BH.

Case II: When $n = 0$, we retrieve the value of energy and angular momentum of extreme GMGHS BH.

Circular motion of the test particle to be exists when both the energy and angular momentum are real and finite. Therefore we must have

$$2r_0^2 - (3r_+ + (2n+1)r_-)r_0 + 2(1+n)r_+r_- > 0 \text{ and } r_0 > r_- . \quad (40)$$

General relativity does not permit arbitrary circular radii, so the denominator of equations (36,37) real only if $2r_0^2 - (3r_+ + (2n+1)r_-)r_0 + 2(1+n)r_+r_- \geq 0$. The limiting case of equality gives an circular orbit with indefinite energy per unit mass, i.e. a circular photon orbit (CPO). This photon orbit is the innermost boundary of the circular orbit for massive particles.

Comparing the above equation of particle orbits with (49) when $r_0 = r_c$, we can observe that photon orbits are the limiting case of time-like circular orbit. It occurs at the radius

$$r_c = r_{ph} = \frac{1}{4} \left[(3r_+ + (2n+1)r_-) \pm \sqrt{(3r_+ + (2n+1)r_-)^2 - 16(1+n)r_+r_-} \right] \quad (41)$$

In the extremal limit, this equation gives

$$r_{ph} = (n+1)r_+ \quad (42)$$

When $n = 0$, we recover the value of CPO of GMGHS BH which is $r_{ph} = r_+ = 2M$ and when $n = 1$, we recover the value of CPO of RN BH which is $r_{ph} = 2r_+ = 2M$.

2.3 Circular Photon Orbits:

For null circular geodesics, the effective potential becomes

$$\mathcal{U}_{eff} = \frac{L^2}{\mathcal{R}^2(r)} \mathcal{F}(r) = \frac{L^2}{r^2} \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right)^{2n-1} \quad (43)$$

In the extremal limit, the effective potential goes to the following form:

$$\mathcal{U}_{eff} = \frac{L^2}{\mathcal{R}^2(r)} \mathcal{F}(r) = \frac{L^2}{r^2} \left(1 - \frac{r_+}{r}\right)^{2n} \quad (44)$$

For circular null geodesics at $r = r_c$, we find

$$\mathcal{U}_{eff} = \mathcal{E}^2 \quad (45)$$

and

$$\frac{d\mathcal{U}_{eff}}{dr} = 0 \quad (46)$$

Thus one may obtain the ratio of energy and angular momentum of the test particle evaluated at $r = r_c$ for circular photon orbits are:

$$\frac{\mathcal{E}_c}{L_c} = \sqrt{\frac{\left(1 - \frac{r_+}{r_c}\right) \left(1 - \frac{r_-}{r_c}\right)^{2n-1}}{r_c^2}} \quad (47)$$

and

$$\mathcal{R}(r_c) \mathcal{F}'(r_c) - 2\mathcal{F}(r_c) \mathcal{R}'(r_c) = 0. \quad (48)$$

or

$$2r_c^2 - (3r_+ + (2n+1)r_-)r_c + 2(1+n)r_+r_- = 0. \quad (49)$$

Let $r_c = r_{ph}$ be the solution of the equation (49) which gives the radius of the circular photon orbit (CPO) of the charged dilation space-time.

Case I: When $n = 1$, we recover the CPO of RN BH.

Case II: When $n = 0$, we retrieve the CPO of GMGHS BH. After incorporating the impact parameter $D_c = \frac{L_c}{E_c}$, the above equation can be written as

$$\frac{1}{D_c} = \frac{\mathcal{E}_c}{L_c} = \sqrt{\frac{\left(1 - \frac{r_+}{r_c}\right) \left(1 - \frac{r_-}{r_c}\right)^{2n-1}}{r_c^2}} \quad (50)$$

At the extremal limit $r_+ = r_-$, this would be

$$\frac{1}{D_c} = \frac{\mathcal{E}_c}{L_c} = \frac{\left(1 - \frac{r_+}{r_c}\right)^n}{r_c} \quad (51)$$

The classical capture cross-section is given by

$$\sigma = \pi D_c^2 = \frac{r_c^2 \left(1 - \frac{r_-}{r_c}\right)^{1-2n}}{\left(1 - \frac{r_+}{r_c}\right)} \quad (52)$$

Case I: When $n = 1$, we recover the classical capture cross-section of RN BH.

Case II: When $n = 0$, we retrieve the classical capture cross-section of GMGHS BH.

The another important class orbit is the marginally bound circular orbit (MBCO) can be found by setting $\mathcal{E}_0^2 = 1$ in Eq. (36), then the MBCO equation reads as

$$2 \left[1 - \left(1 - \frac{r_+}{r_0}\right)^2 \left(1 - \frac{r_-}{r_0}\right)^{n+1} \right] r_0^2 - \left[3r_+ + \{(2n+1) - (1-n) \left(1 - \frac{r_+}{r_0}\right)^2 \left(1 - \frac{r_-}{r_0}\right)^n\} r_- \right] r_0 + 2(1+n)r_+r_- = 0 \quad (53)$$

Case I: When $n = 1$, we retrieve the MBCO of RN BH, which may be determined from the following equation:

$$(r_+ + r_-)r_0^3 - 2(r_+ + r_-)^2r_0^2 + 4r_+r_-(r_+ + r_-)r_0 - 2(r_+r_-)^2 = 0 \quad (54)$$

Let $r_0 = r_{mb}$ be the solution of the equation which gives the radius of MBCO of RN BH.

Case II: When $n = 0$, we retrieve the MBCO of GMGHS BH, which can be determined from the following equation:

$$r_0^2 - 2r_+r_0 + r_+r_- = 0 \quad (55)$$

or

$$r_0 = r_{mb} = r_+ \pm \sqrt{r_+(r_+ - r_-)} \quad (56)$$

From the astrophysical point of view, the most important class of orbit is the ISCO which may be derived from the second derivative of the effective potential of time-like case. i.e.

$$\frac{d^2\mathcal{V}_{eff}}{dr^2} = 0 \quad (57)$$

Thus one may obtain the ISCO equation for the charge dilation space-time from the following functional equation:

$$\begin{aligned} & \mathcal{F}(r_0)\mathcal{F}''(r_0)\mathcal{R}(r_0)\mathcal{R}'(r_0) - \mathcal{F}(r_0)\mathcal{F}'(r_0)\mathcal{R}(r_0)\mathcal{R}''(r_0) - \\ & 2\mathcal{R}(r_0)\mathcal{R}'(r_0)(\mathcal{F}'(r_0))^2 - 3\mathcal{F}(r_0)\mathcal{F}'(r_0)(\mathcal{R}'(r_0))^2 = 0 . \end{aligned} \quad (58)$$

For extremal case ($r_+ = r_-$), it can be easily obtained the ISCO equation for charged dilation BH which is given by

$$r_0^2 - (3n + 2)r_+r_0 + (n + 1)^2r_+^2 = 0 \quad (59)$$

or

$$r_0 = \frac{r_+}{2} \left[(3n + 2) \pm \sqrt{n(5n + 4)} \right] \quad (60)$$

In the limit $n \rightarrow 1$, we obtain the radius of ISCO for RN BH, which is $r_0 = r_{ISCO} = 4M$ and in the limit $n \rightarrow 0$, we recover the radius of ISCO for GMGHS BH which is $r_0 = r_{ISCO} = 2M$.

3 CM Energy of Particle Collision near the horizon of the Dilation BH:

This section is devoted to study the particle acceleration and collision in the CM frame. To compute the CM energy, we consider two particles coming from infinity with $\frac{\mathcal{E}_1}{m_0} = \frac{\mathcal{E}_2}{m_0} = 1$ approaching the charged dilation BH with different angular momenta L_1 and L_2 and colliding at some radius r . Later, we consider the collision point r to approach the horizon $r = r_+$. Also we have assumed the the particles to be at rest at infinity.

The CM energy is evaluated by using the following formula which was first given by BSW [1] reads

$$\left(\frac{\mathcal{E}_{cm}}{\sqrt{2}m_0} \right)^2 = 1 - g_{\mu\nu}u_1^\mu u_2^\nu . \quad (61)$$

We shall also assume throughout this work the geodesic motion of the colliding particles confined in the $\theta = \frac{\pi}{2}$ plane. Since the charged dilation space-time has a time-like isometry followed by the time-like Killing vector field χ whose projection along the four velocity \mathbf{u} of geodesics $\chi \cdot \mathbf{u} = -\mathcal{E}$, is conserved along such geodesics (where $\chi \equiv \partial_t$). Similarly there is also the ‘angular momentum’ $L = \zeta \cdot \mathbf{u}$ is conserved due to the rotational symmetry (where $\zeta \equiv \partial_\phi$).

For massive particles, the components of the four velocity are

$$u^t = \dot{t} = \frac{\mathcal{E}}{\mathcal{F}(r)} \quad (62)$$

$$u^r = \dot{r} = \pm \sqrt{\mathcal{E}^2 - \mathcal{F}(r) \left(1 + \frac{L^2}{\mathcal{R}^2(r)}\right)} \quad (63)$$

$$u^\theta = \dot{\theta} = 0 \quad (64)$$

$$u^\phi = \dot{\phi} = \frac{L}{\mathcal{R}^2(r)} . \quad (65)$$

and

$$u_1^\mu = \left(\frac{\mathcal{E}_1}{\mathcal{F}(r)}, -X_1, 0, \frac{L_1}{\mathcal{R}^2(r)} \right) . \quad (66)$$

$$u_2^\mu = \left(\frac{\mathcal{E}_2}{\mathcal{F}(r)}, -X_2, 0, \frac{L_2}{\mathcal{R}^2(r)} \right) . \quad (67)$$

Substituting this in (61), we get the center of mass energy:

$$\left(\frac{\mathcal{E}_{cm}}{\sqrt{2}m_0} \right)^2 = 1 + \frac{\mathcal{E}_1\mathcal{E}_2}{\mathcal{F}(r)} - \frac{X_1X_2}{\mathcal{F}(r)} - \frac{L_1L_2}{\mathcal{R}^2(r)} . \quad (68)$$

where,

$$X_1 = \sqrt{\mathcal{E}_1^2 - \mathcal{F}(r) \left(1 + \frac{L_1^2}{\mathcal{R}^2(r)}\right)}, \quad X_2 = \sqrt{\mathcal{E}_2^2 - \mathcal{F}(r) \left(1 + \frac{L_2^2}{\mathcal{R}^2(r)}\right)}$$

For simplicity, $\mathcal{E}_1 = \mathcal{E}_2 = 1$ and substituting the value of $\mathcal{F}(r)$, we obtain the CM energy near the event horizon (r_+) of the charged dilation space-time:

$$\mathcal{E}_{cm} |_{r \rightarrow r_+} = \sqrt{2}m_0 \sqrt{\frac{4\mathcal{R}_+^2 + (L_1 - L_2)^2}{2\mathcal{R}_+^2}} . \quad (69)$$

where r_+ is described in equation (8). Putting the value of $\mathcal{R}_+^2 = r_+^{1+n}(r_+ - r_-)^{1-n}$ in the above equation we get

$$\mathcal{E}_{cm} |_{r \rightarrow r_+} = \sqrt{2}m_0 \sqrt{\frac{4r_+^{n+1}(r_+ - r_-)^{1-n} + (L_1 - L_2)^2}{2r_+^{n+1}(r_+ - r_-)^{1-n}}} . \quad (70)$$

Which shows that the CM energy is finite and depends upon the values of the angular momentum parameter.

Whenever we taking the extremal limit $r_+ = r_-$, we get the CM energy near the event horizon $r = r_+$:

$$\mathcal{E}_{cm} \big|_{r \rightarrow r_+} \mapsto \infty \quad (71)$$

which suggests that the CM energy of collision for extremal charged dilation BH is diverging as we approaches the extremal limit. Thus we get the unlimited CM energies. Interestingly, it is independent of the fine tuning of the angular momentum parameter. This is one of the key point of the paper.

Case I: When $n = 1$, we recover the value of CM energy of RN BH, which is given by

$$\mathcal{E}_{cm} \big|_{r \rightarrow r_+} = \sqrt{2}m_0 \sqrt{\frac{4r_+^2 + (L_1 - L_2)^2}{2r_+^2}}. \quad (72)$$

Case II: When $n = 0$, we retrieve the value of CM energy of GMGHS BH which is given by

$$\mathcal{E}_{cm} = \sqrt{2}m_0 \sqrt{\frac{8M(2M - b) + (L_1 - L_2)^2}{4M(2M - b)}} \quad (73)$$

In the limit $b \rightarrow 0$, the above expression reduces to

$$\mathcal{E}_{cm} = \sqrt{2}m_0 \sqrt{\frac{16M^2 + (L_1 - L_2)^2}{8M^2}}. \quad (74)$$

which is the CM energy of the Schwarzschild BH. In fact this is indeed a finite quantity first observed in [1].

Interestingly, near the Cauchy horizon (r_-) the CM energy for charged dilation space-time is found to be

$$\mathcal{E}_{cm} \big|_{r \rightarrow r_-} = \sqrt{2}m_0 \sqrt{\frac{4\mathcal{R}_-^2 + (L_1 - L_2)^2}{2\mathcal{R}_-^2}}. \quad (75)$$

where r_- is described in equation (9).

Since the values of \mathcal{R}_-^2 is zero, i.e. $\mathcal{R}_-^2 = 0$, thus the value of CM energy near the inner horizon leads to diverging value. Hence,

$$\mathcal{E}_{cm} \big|_{r \rightarrow r_-} \mapsto \infty \quad (76)$$

This is another interesting feature of the charged dilation BH.

4 CM Energy of Particle Collision near the ISCO of an Extremal RN BH:

The aim of this section is to compute the CM energy for RN BH at $r = r_{ISCO}$, $r = r_{mb}$ and $r = r_+$. Then we compare the results obtained for the different collision point. The metric for RN space-time can be written as

$$ds^2 = -\mathcal{G}(r)dt^2 + \frac{dr^2}{\mathcal{G}(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) . \quad (77)$$

where the function $\mathcal{G}(r)$ is defined by

$$\mathcal{G}(r) = \left(1 - \frac{r_+}{r}\right) \left(1 - \frac{r_-}{r}\right) . \quad (78)$$

The BH has event horizon which is situated at $r_+ = M + \sqrt{M^2 - Q^2}$ and Cauchy horizon which is situated at $r_- = M - \sqrt{M^2 - Q^2}$. The equatorial time-like geodesics of RN BHs are (See Figs. 1-2)

$$u^t = \frac{\mathcal{E}}{\mathcal{G}(r)} \quad (79)$$

$$u^r = \pm \sqrt{\mathcal{E}^2 - \mathcal{G}(r) \left(1 + \frac{L^2}{r^2}\right)} \quad (80)$$

$$u^\theta = 0 \quad (81)$$

$$u^\phi = \frac{L}{r^2} . \quad (82)$$

Next, we compute the CM energy for the RN space-time by using the well known formula as given by Bañados et al. in [1]:

$$\left(\frac{\mathcal{E}_{cm}}{\sqrt{2}m_0}\right)^2 = 1 - g_{\mu\nu}u_1^\mu u_2^\nu . \quad (83)$$

where u_1^μ and u_2^ν are the 4-velocity of the two particles, which can be found from the following equation(82).

$$u_1^\mu = \left(\frac{\mathcal{E}_1}{\mathcal{G}(r)}, -Y_1, 0, \frac{L_1}{r^2}\right) . \quad (84)$$

$$u_2^\mu = \left(\frac{\mathcal{E}_2}{\mathcal{G}(r)}, -Y_2, 0, \frac{L_2}{r^2}\right) . \quad (85)$$

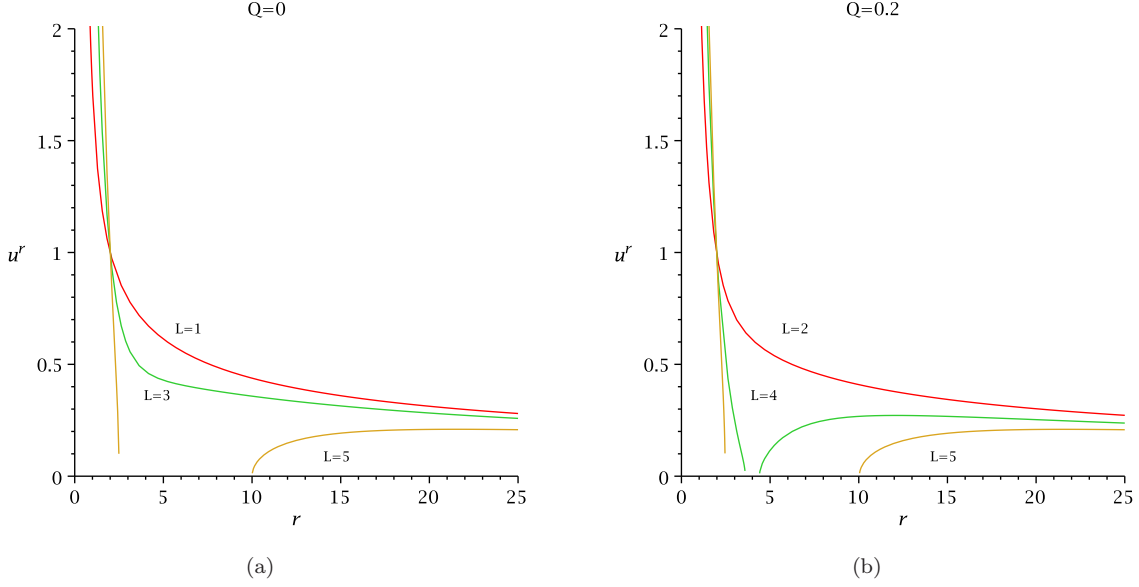


Figure 1: The figure shows the variation of \dot{r} with r for RN BH with $Q = 0$ and $Q = 0.2$.

Therefore using(83) one can obtain the center-of-mass energy for this collision:

$$\left(\frac{\mathcal{E}_{cm}}{\sqrt{2}m_0}\right)^2 = 1 + \frac{\mathcal{E}_1\mathcal{E}_2}{\mathcal{G}(r)} - \frac{Y_1Y_2}{\mathcal{G}(r)} - \frac{L_1L_2}{r^2}. \quad (86)$$

$$\text{where} \quad (87)$$

$$Y_1 = \sqrt{\mathcal{E}_1^2 - \mathcal{G}(r) \left(1 + \frac{L_1^2}{r^2}\right)}$$

$$Y_2 = \sqrt{\mathcal{E}_2^2 - \mathcal{G}(r) \left(1 + \frac{L_2^2}{r^2}\right)} \quad (88)$$

As we have assumed $\mathcal{E}_1 = \mathcal{E}_2 = 1$ previously, and substituting the value of $\mathcal{G}(r)$ one could obtain finally the CM energy near the event horizon(r_+) for non-extremal RN space-time(See Figs. 3-4)

$$\mathcal{E}_{cm} |_{r \rightarrow r_+} = \sqrt{2}m_0 \sqrt{\frac{4r_+^2 + (L_1 - L_2)^2}{2r_+^2}}. \quad (89)$$

Again we have already known from [1], the maximum CM energy strictly depend upon the values of critical angular momentum such that the particles can reach the event horizon with maximum tangential velocity. Thus the critical angular momentum and the

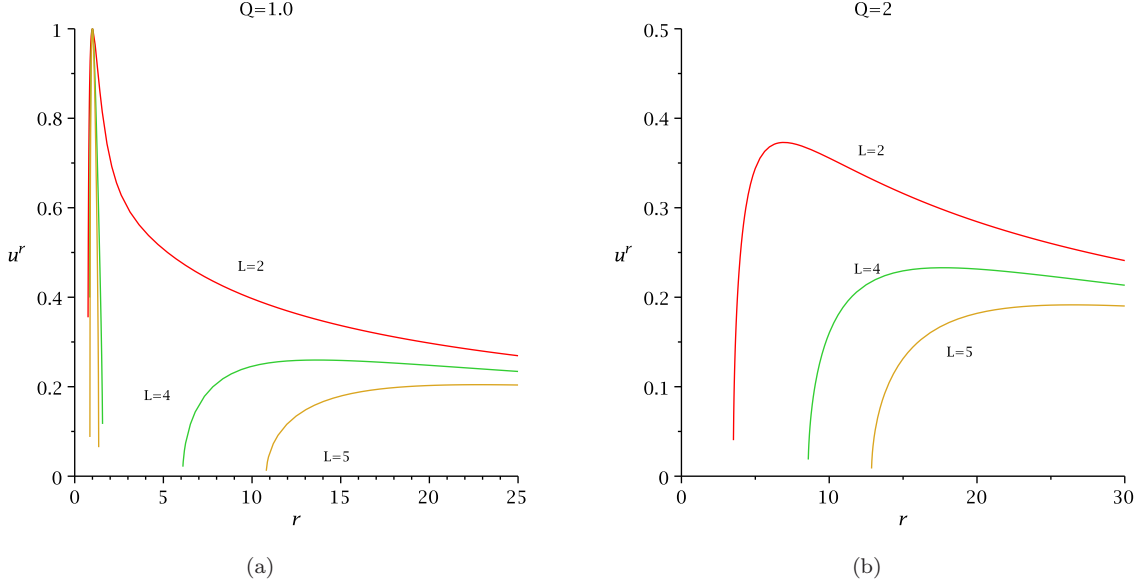


Figure 2: The figure shows the variation of \dot{r} with r for RN BH with $Q = 1.0$ and $Q = 2.0$.

critical radius could be calculated from the radial effective potential. Now we impose the following criterion for determining the critical values of angular momentum i.e. :

$$\dot{r}^2 r^4 = (\mathcal{E}^2 - 1)r^4 + 2Mr^3 - (L^2 + Q^2)r^2 + 2ML^2r - Q^2L^2 = 0 \quad (90)$$

Now setting $\mathcal{E}^2 = 1$ for marginal case, the equation turns out to be

$$2Mr^3 - (L^2 + Q^2)r^2 + 2ML^2r - Q^2L^2 = 0. \quad (91)$$

For non-extremal RN BH, the critical values of angular momentum can be determined by finding the real root of the above equation and using the following condition:

$$(M^2 - Q^2)L^6 - (3Q^4 - 20M^2Q^2 + 16M^4)L^4 - (8M^2 + 3Q^2)Q^4L^2 - Q^8 = 0 \quad (92)$$

It can be easily seen that in the limit $Q = 0$, we retrieve the critical values of angular momentum for Schwarzschild BH:

$$L = \pm 4M \quad (93)$$

Since, we are interested in this work for extremal RN BH, thus it can be easily calculated the critical values of angular momentum parameter in the extremal limit $M = Q$. Therefore the range of L for in-falling geodesics is

$$-\sqrt{\frac{11 + 5\sqrt{5}}{2}}M \leq L \leq \sqrt{\frac{11 + 5\sqrt{5}}{2}}M \quad (94)$$

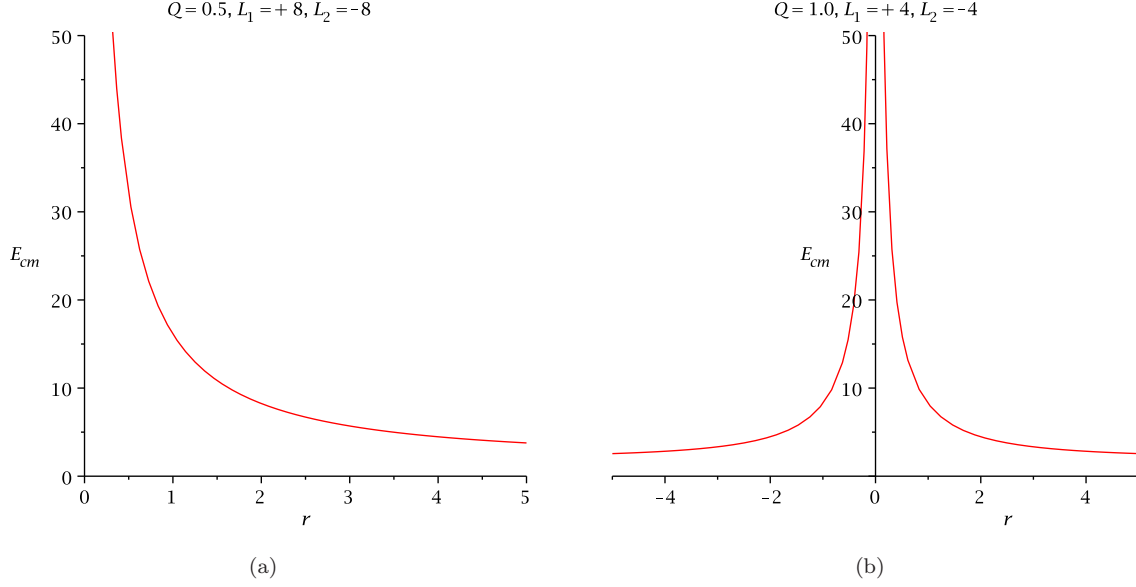


Figure 3: The figure shows the variation of \mathcal{E}_{cm} with r for RN BH with $Q = 0.5$ and $L_1 = 8, L_2 = -8$, $Q = 1.0$ and $L_1 = 4, L_2 = -4$

For extremal RN space-time, the values of CM energy near the horizon is given by

$$\mathcal{E}_{cm} |_{r \rightarrow M} = \sqrt{2}m_0 \sqrt{\frac{4M^2 + (L_1 - L_2)^2}{2M^2}}. \quad (95)$$

Since here we have assumed that the collision point is at $r = M$ and two particles have the angular momentum which is given by the following equation:

$$L = \pm \sqrt{\frac{r^2(2Mr - Q^2)}{r^2 - 2Mr + Q^2}} \quad (96)$$

In the extremal limit it goes to

$$L = \pm \sqrt{\frac{Mr^2(2r - M)}{(r - M)^2}} \quad (97)$$

It was already mentioned in [1], a new phenomenon would appear if one of the particles participating in the collision has the critical angular momentum. If one of the particles have the diverging angular momentum at the horizon i.e.

$$L_1 |_{r=M} = \sqrt{\frac{Mr^2(2r - M)}{(r - M)^2}} \rightarrow \infty \quad (98)$$

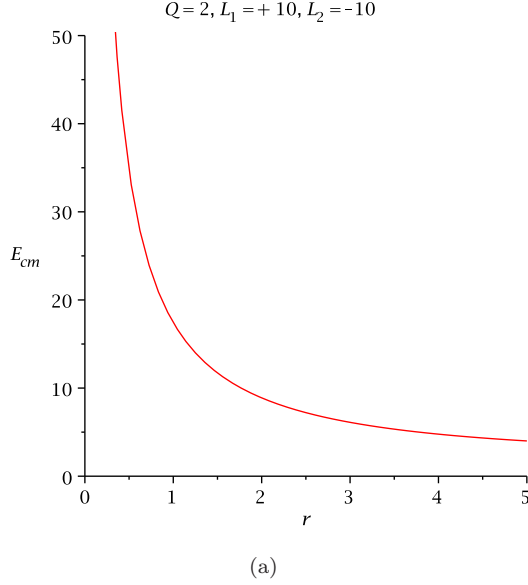


Figure 4: The figure shows the variation of \mathcal{E}_{cm} with r for RN BH with $Q = 2.0$ and $L_1 = 10, L_2 = -10$.

then the corresponding value of the CM energy (using Eq. 95) for extremal RN BH at the extremal horizon is

$$\mathcal{E}_{cm} |_{r \rightarrow M} = \sqrt{2}m_0 \sqrt{\frac{4M^2 + (L_1 - L_2)^2}{2M^2}} \rightarrow \infty \quad (99)$$

Thus for the extremal RN BH, we get the infinite amounts of CM energy.

Now we will see what happens the CM energy if we choose the collision point to be ISCO. It is well known that for extremal RN BH the ISCO is at $r = 4M$ [28]. Now if we consider the collision point to be ISCO then the corresponding value of CM energy at the ISCO for extremal RN space-time is given by

$$\mathcal{E}_{cm} |_{r \rightarrow 4M} = \sqrt{2}m_0 \sqrt{\frac{400M^2 - 9L_1L_2 - \sqrt{112M^2 - 9L_1^2}\sqrt{112M^2 - 9L_2^2}}{144M^2}}. \quad (100)$$

For the critical values of angular momentum $L_1 = \sqrt{\frac{11+5\sqrt{5}}{2}}M$ and $L_2 = -\sqrt{\frac{11+5\sqrt{5}}{2}}M$, the CM energy is found to be

$$\mathcal{E}_{cm} |_{r \rightarrow 4M} = \sqrt{2}m_0 \sqrt{\frac{387 + 45\sqrt{5}}{144}} = 2.60m_0 \quad (101)$$

Similarly, one may compute the MBCO for RN BH by setting $\mathcal{E}_0^2 = 1$ which turns out to be $r_{mb} = \left(\frac{3+\sqrt{5}}{2}\right)M$. Thus when the collision point to be MBCO, the CM energy is found to be at $r = r_{mb}$:

$$\mathcal{E}_{cm} \big|_{r \rightarrow r_{mb}} = \sqrt{2}m_0 \sqrt{\frac{(10 + 4\sqrt{5})(7 + 3\sqrt{5})M^2 - (6 + 2\sqrt{5})L_1L_2 - \Lambda_1\Lambda_2}{(3 + \sqrt{5})(7 + 3\sqrt{5})M^2}}. \quad (102)$$

where

$$\Lambda_1 = \sqrt{(58 + 26\sqrt{5})M^2 - (6 + 2\sqrt{5})L_1^2} \quad (103)$$

$$\Lambda_2 = \sqrt{(58 + 26\sqrt{5})M^2 - (6 + 2\sqrt{5})L_2^2} \quad (104)$$

For the critical values of angular momentum $L_1 = \sqrt{\frac{11+5\sqrt{5}}{2}}M$ and $L_2 = -\sqrt{\frac{11+5\sqrt{5}}{2}}M$, the CM energy is at r_{mb} found to be

$$\mathcal{E}_{cm} \big|_{r=r_{mb}} = \sqrt{2}m_0 \sqrt{\frac{47 + 21\sqrt{5}}{9 + 4\sqrt{5}}} = 3.23m_0 \quad (105)$$

Now we can compare the results obtained as

$$\mathcal{E}_{cm} \big|_{r_+=M} : \mathcal{E}_{cm} \big|_{r_{mb}=\left(\frac{3+\sqrt{5}}{2}\right)M} : \mathcal{E}_{cm} \big|_{r_{ISCO}=4M} = \infty : 3.23 : 2.6 \quad (106)$$

Thus we get,

$$\mathcal{E}_{cm} \big|_{r_+} > \mathcal{E}_{cm} \big|_{r_{mb}} > \mathcal{E}_{cm} \big|_{r_{ISCO}} \quad (107)$$

It is clearly evident that CM energy is diverging at the event horizon and finite at the MBCO and at ISCO.

In the limit $L_1 = L_2 = 0$, we obtain the following equality:

$$\mathcal{E}_{cm} \big|_{r_+} = \mathcal{E}_{cm} \big|_{r_{mb}} = \mathcal{E}_{cm} \big|_{r_{ISCO}} = 2m_0 \quad (108)$$

5 CM Energy of Particle Collision near the ISCO of a Schwarzschild BH:

Here, we shall extend our analysis for Schwarzschild BH and calculate the CM energy of the colliding particles at various collision points. First we choose the collision point to be horizon, then we choose the collision point to be ISCO and finally we have chosen the

collision point is at MBO. To proceed, we first write the metric of the Schwarzschild BH in Schwarzschild coordinates are

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) . \quad (109)$$

The BH has an event horizon located at $r = 2M$. For this BH, the equatorial time-like geodesics are described by

$$u^t = \frac{\mathcal{E}}{1 - \frac{2M}{r}} \quad (110)$$

$$u^r = \pm \sqrt{\mathcal{E}^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L^2}{r^2}\right)} \quad (111)$$

$$u^\theta = 0 \quad (112)$$

$$u^\phi = \frac{L}{r^2} . \quad (113)$$

and the components of four velocity are (See Fig. 5a)

$$u_1^\mu = \left(\frac{\mathcal{E}_1}{1 - \frac{2M}{r}}, -Z_1, 0, \frac{L_1}{r^2} \right) . \quad (114)$$

$$u_2^\mu = \left(\frac{\mathcal{E}_2}{1 - \frac{2M}{r}}, -Z_2, 0, \frac{L_2}{r^2} \right) . \quad (115)$$

Since the Schwarzschild space-time is the special case of RN space-time, therefore one can compute the CM energy (See Figs. 5b-7) using the equation (86) as

$$\left(\frac{\mathcal{E}_{cm}}{\sqrt{2}m_0} \right)^2 = 1 + \frac{\mathcal{E}_1 \mathcal{E}_2}{\left(1 - \frac{2M}{r}\right)} - \frac{Z_1 Z_2}{\left(1 - \frac{2M}{r}\right)} - \frac{L_1 L_2}{r^2} . \quad (116)$$

$$\text{where} \quad (117)$$

$$\begin{aligned} Z_1 &= \sqrt{\mathcal{E}_1^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L_1^2}{r^2}\right)} \\ Z_2 &= \sqrt{\mathcal{E}_2^2 - \left(1 - \frac{2M}{r}\right) \left(1 + \frac{L_2^2}{r^2}\right)} \end{aligned} \quad (118)$$

Setting $\mathcal{E}_1 = \mathcal{E}_2 = 1$ as is, one may obtain the CM energy near the event horizon for Schwarzschild BH (first calculated in [1]) is given by

$$\mathcal{E}_{cm} |_{r \rightarrow 2M} = \sqrt{2}m_0 \sqrt{\frac{16M^2 + (L_1 - L_2)^2}{8M^2}} . \quad (119)$$

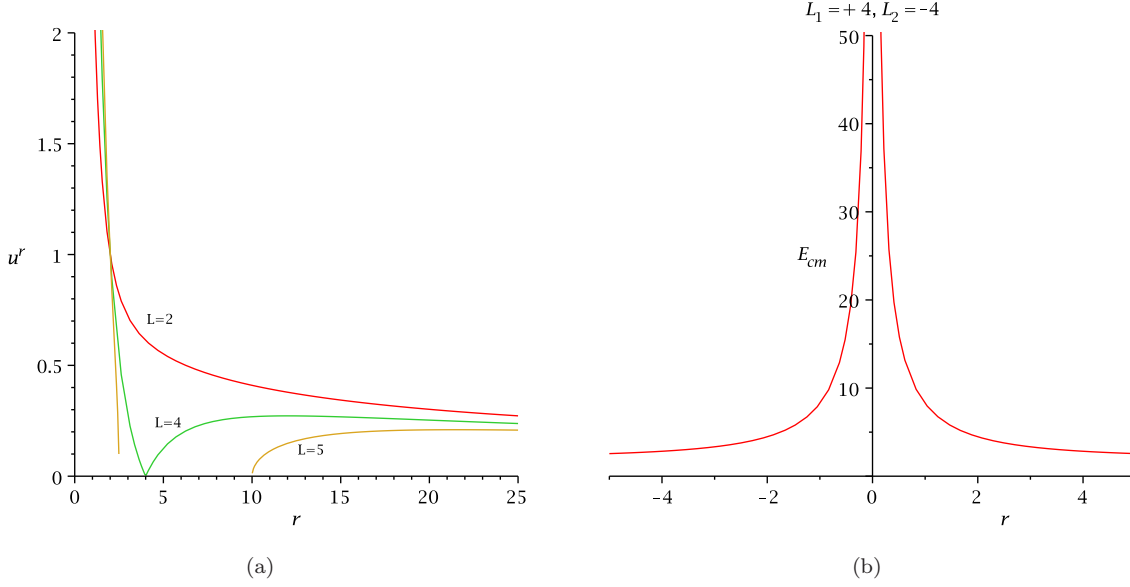


Figure 5: The LHS figure shows the variation of \dot{r} with r for Schwarzschild BH and The RHS figure shows the variation of \mathcal{E}_{cm} with r for Schwarzschild BH.

When $L_1 = 4M$ and $L_2 = -4M$, the maximum CM energy for Schwarzschild BH is found to be $\mathcal{E}_{cm} |_{r \rightarrow 2M} = 2\sqrt{5}m_0$.

Again, we know the ISCO for Schwarzschild BH is $r = 6M$, thus the CM energy at ISCO is calculated to be

$$\mathcal{E}_{cm} |_{r \rightarrow 6M} = \sqrt{2}m_0 \sqrt{\frac{(90M^2 - L_1 L_2) - \sqrt{18M^2 - L_1^2} \sqrt{18M^2 - L_2^2}}{36M^2}}. \quad (120)$$

For the critical values of angular momentum $L_1 = 4M$ and $L_2 = -4M$, the CM energy is given by

$$\mathcal{E}_{cm} |_{r \rightarrow r_{ISCO}=6M} = \sqrt{\frac{52}{9}}m_0. \quad (121)$$

It is well known that the MBCO for Schwarzschild BH is occur at $r_{mb} = 4M$, thus the CM energy is found to be:

$$\mathcal{E}_{cm} |_{r \rightarrow 4M} = \sqrt{2}m_0 \sqrt{\frac{(48M^2 - L_1 L_2) - \sqrt{16M^2 - L_1^2} \sqrt{16M^2 - L_2^2}}{16M^2}}. \quad (122)$$

If we take $L_1 = L_2 = 0$, the CM energy is found to be $\mathcal{E}_{cm} = 2m_0$. Whenever we take $L_1 = 4M$ and $L_2 = -4M$, we have the CM energy

$$\mathcal{E}_{cm} |_{r \rightarrow r_{mb}=4M} = 2\sqrt{2}m_0. \quad (123)$$

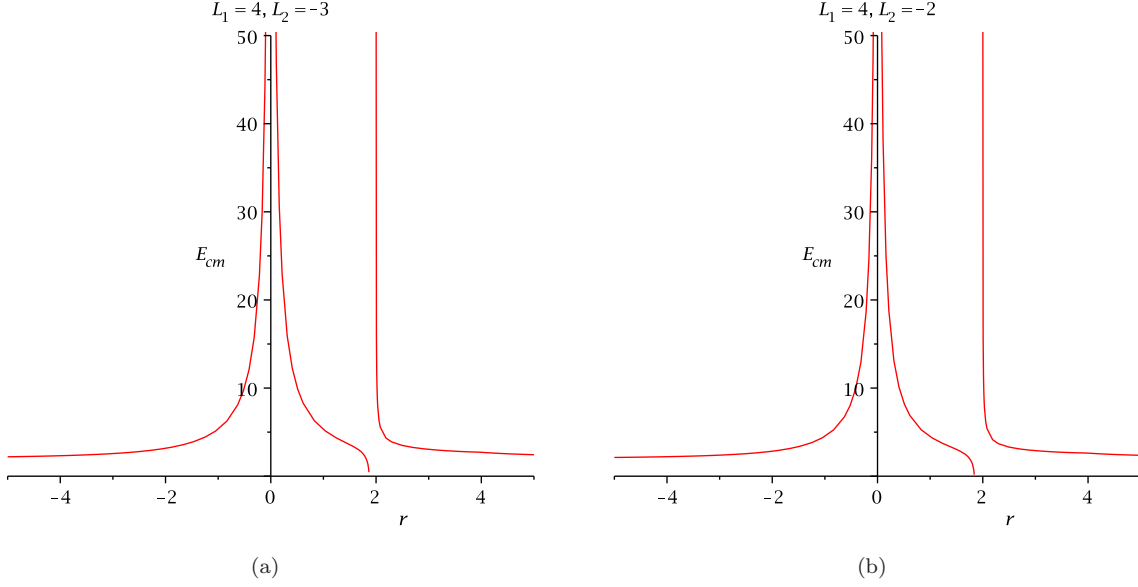


Figure 6: The figures show the variation of \mathcal{E}_{cm} with r for Schwarzschild BH.

Now we may compare the results obtained as

$$\mathcal{E}_{cm} \big|_{r_+=2M} : \mathcal{E}_{cm} \big|_{r_{mb}=4M} : \mathcal{E}_{cm} \big|_{r_{ISCO}=6M} = 2\sqrt{5} : 2\sqrt{2} : \sqrt{\frac{52}{9}} \quad (124)$$

$$= \sqrt{5} : \sqrt{2} : \frac{\sqrt{13}}{3} \quad (125)$$

$$= 2.23 : 1.41 : 1.20 \quad (126)$$

Thus we have found,

$$\mathcal{E}_{cm} \big|_{r_+} > \mathcal{E}_{cm} \big|_{r_{mb}} > \mathcal{E}_{cm} \big|_{r_{ISCO}} \quad (127)$$

It is clearly evident that CM energy is maximum at the event horizon than the MBCO and ISCO.

6 CM energy of Particle collision near the horizon of the GMGHS BH:

In an earlier paper [30], we showed that a string BHs may act as a particle accelerators to arbitrarily high center-of-mass energy. Also, we proved that for extremal GMGHS space-time the center of mass energy of collision at $r \equiv r_{ISCO} = r_{ph} = r_{mb} = r_{hor} = 2M$ is

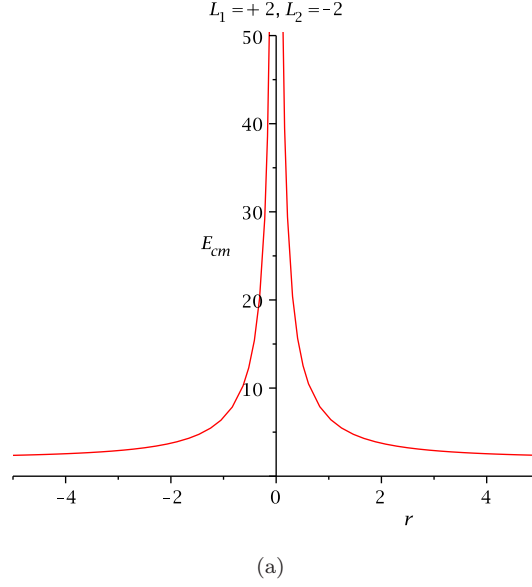


Figure 7: The figure shows the variation of \mathcal{E}_{cm} with r for Schwarzschild BH.

arbitrarily large. Here, we would like to discuss the same more elaborately and graphically which was not given in the previous work.

To proceed it, first we consider the time-like geodesics(See Figs.8-9) on the equatorial plane [30] reads

$$u^t = \frac{\mathcal{E}}{\mathcal{H}(r)} \quad (128)$$

$$u^r = \pm \sqrt{\mathcal{E}^2 - \mathcal{H}(r) \left(1 + \frac{L^2}{r(r-b)} \right)} \quad (129)$$

$$u^\theta = 0 \quad (130)$$

$$u^\phi = \frac{L}{r(r-b)} . \quad (131)$$

where $\mathcal{H}(r) = 1 - \frac{2M}{r}$ and $b = \frac{Q^2}{M} e^{-2\phi_0}$. and

$$u_1^\mu = \left(\frac{\mathcal{E}_1}{\mathcal{H}(r)}, -X_1, 0, \frac{L_1}{r^2} \right) . \quad (132)$$

$$u_2^\mu = \left(\frac{\mathcal{E}_2}{\mathcal{H}(r)}, -X_2, 0, \frac{L_2}{r^2} \right) . \quad (133)$$

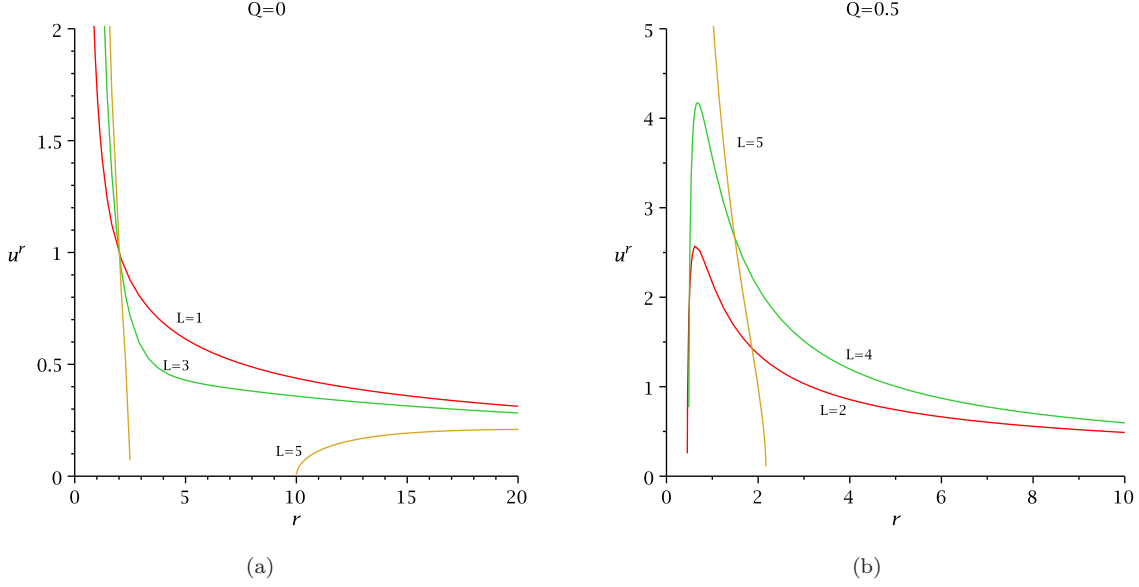


Figure 8: The figure shows the variation of \dot{r} with r for GMGHS BH.

Therefore using(61), we obtain the CM energy(See Figs. 10-11) for this collision:

$$\left(\frac{\mathcal{E}_{cm}}{\sqrt{2}m_0}\right)^2 = 1 + \frac{\mathcal{E}_1\mathcal{E}_2}{\mathcal{H}(r)} - \frac{X_1X_2}{\mathcal{H}(r)} - \frac{L_1L_2}{r(r-b)}. \quad (134)$$

where

$$\begin{aligned} X_1 &= \sqrt{\mathcal{E}_1^2 - \mathcal{H}(r) \left(1 + \frac{L_1^2}{r(r-b)}\right)} \\ X_2 &= \sqrt{\mathcal{E}_2^2 - \mathcal{H}(r) \left(1 + \frac{L_2^2}{r(r-b)}\right)} \end{aligned} \quad (136)$$

As we have assumed $\mathcal{E}_1 = \mathcal{E}_2 = 1$ previously and substituting $\mathcal{H}(r) = 1 - \frac{2M}{r}$, we obtain finally the CM energy near the horizon:

$$\mathcal{E}_{cm} = \sqrt{2}m_0 \sqrt{\frac{8M(2M-b) + (L_1 - L_2)^2}{4M(2M-b)}}. \quad (137)$$

Whenever we taking the extremal limit $b = 2M$, we get the CM energy near the horizon:

$$\mathcal{E}_{cm} = \sqrt{2}m_0 \sqrt{\frac{8M(2M-b) + (L_1 - L_2)^2}{4M(2M-b)}} \quad (138)$$

$$\mathcal{E}_{cm} \mapsto \infty \quad (139)$$

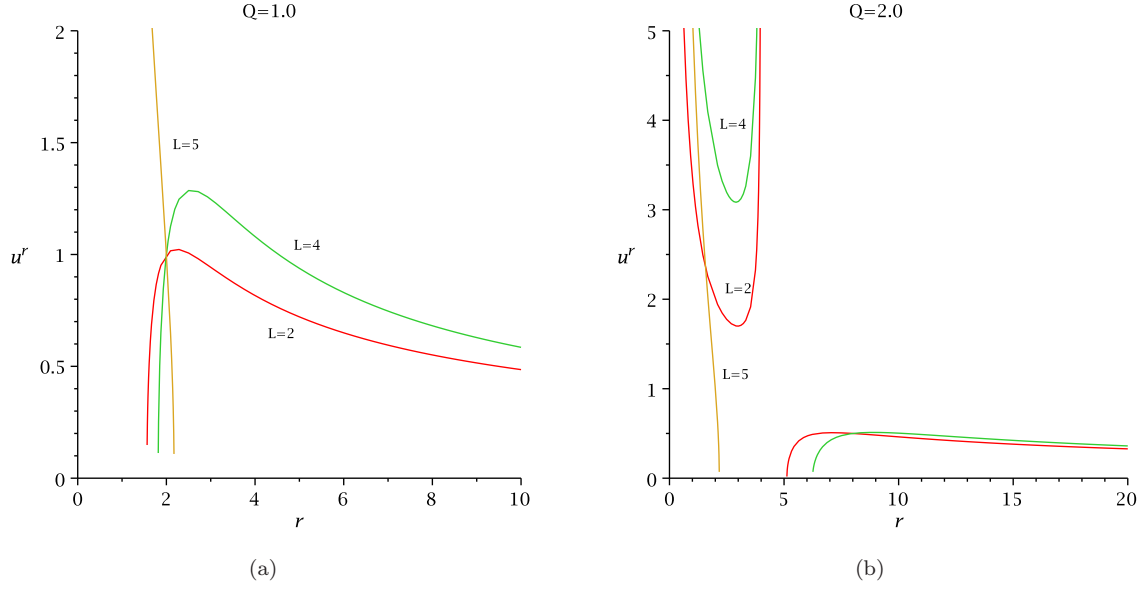


Figure 9: The figure shows the variation of \dot{r} with r for GMGHS BH.

which implies that the CM energy of collision for extremal dilation black hole blows up as we approach the extremal limit.

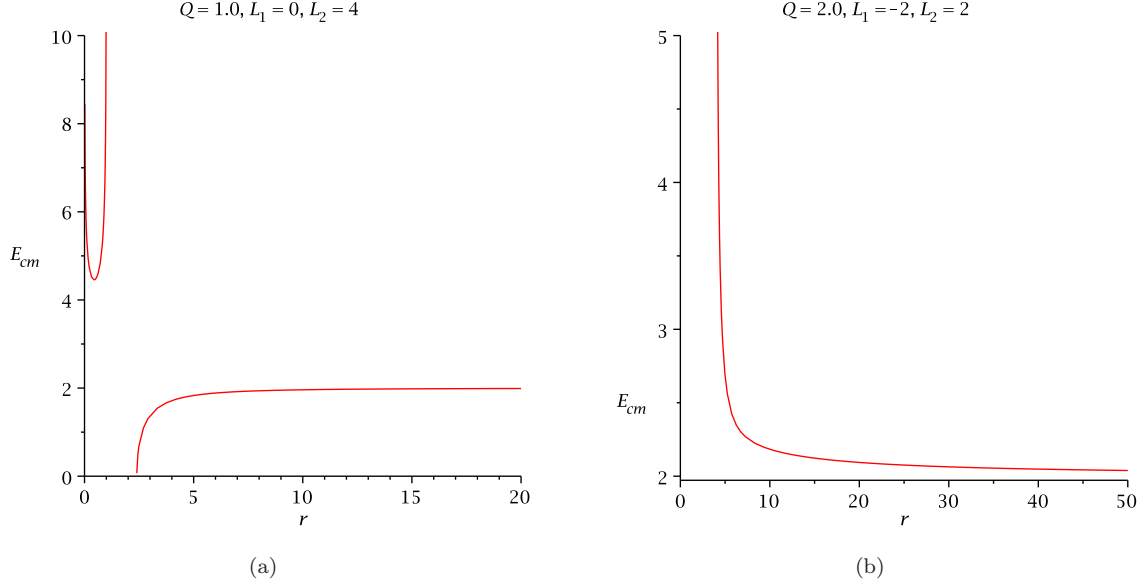


Figure 10: The figure shows the variation of \mathcal{E}_{cm} with r for GMGHS BH.

Again, it is well known that for extremal GMGHS space-time [29] the ISCO, circular photon orbit(CPO) and marginally bound circular orbit(MBCO) coincide with the same radius i.e. $r_{ISCO} = r_{ph} = r_{mb} = 2M$. If we choose the different collision point say ISCO or CPO or MBCO, then the CM energy will be unlimited for each collision point. Now we have compared the \mathcal{E}_{cm} for different collision point:

$$\mathcal{E}_{cm} |_{r_+=2M} : \mathcal{E}_{cm} |_{r_{mb}=2M} : \mathcal{E}_{cm} |_{r_{ISCO}=2M} = \infty : \infty : \infty \quad (140)$$

Since for this extreme BH, the collision point is at same location thus the CM energy gives the diverging value at each collision point which is quite different from extreme RN BH and Schwarzschild BH.

7 Summary and Outlook:

In this paper, we have performed the collision of two neutral particles falling freely from rest at infinity in the background of the charged dilation BH. Firstly, we have studied the complete geodesic structure of the charged dilation BHs both time-like case and null case respectively. ISCO, MBCO and CPO of the said BHs also have been computed.

Then we have discussed the extremal cases for both massive particles and massless particles. Next, we have calculated the CM energy in the center-of-mass frame for charged dilation BHs. We have found that the \mathcal{E}_{cm} is diverging for the extremal situation, whereas \mathcal{E}_{cm} is finite for the non-extremal situation. There are two distinct outcome we have

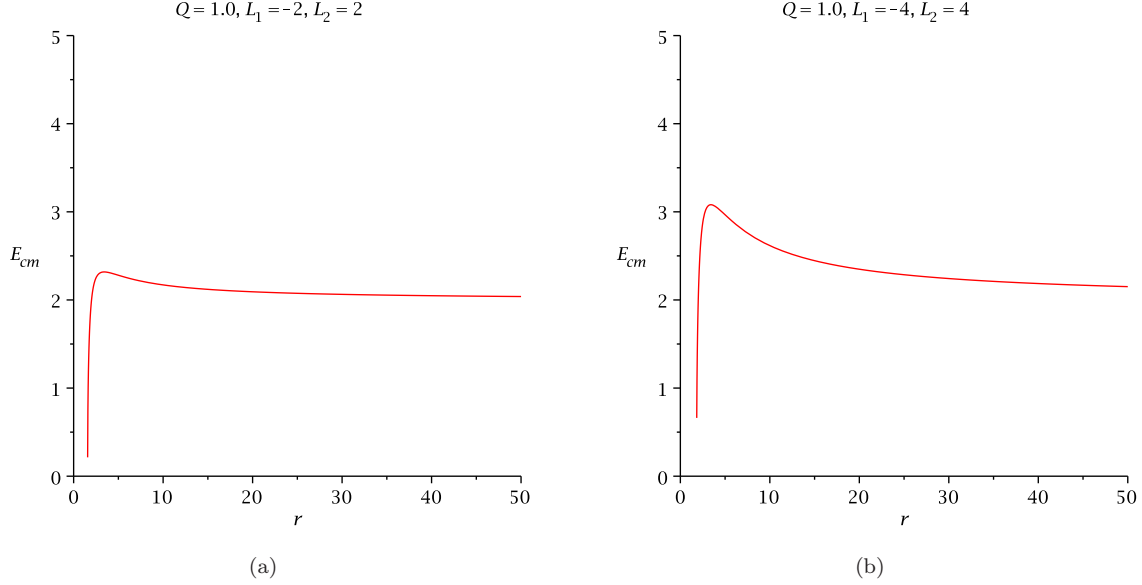


Figure 11: The figure shows the variation of \mathcal{E}_{cm} with r for GMGHS BH.

seen. When $n = 1$, we recover the value of \mathcal{E}_{cm} for RN BH and for extremal case, it gives the diverging value. Again, when $n = 1$, we retrieve the value of \mathcal{E}_{cm} for GMGHS BH. Interestingly, for this extreme BH we showed that in [29] the ISCO, photon orbit and marginally bound circular orbit were coincident with the event horizon i.e $r_{ISCO} = r_{ph} = r_{mb} = r_+ = 2M$. Consequently, the \mathcal{E}_{cm} at $r \equiv r_{ISCO} = r_{ph} = r_{mb} = r_{hor} = 2M$ gives the diverging value [30]. Which is completely different from extreme RN BH, where, $r_{ISCO} \neq r_{ph} \neq r_{mb} \neq r_+ = M$.

We also showed that for extreme RN BH the CM energy is diverging at the extremal horizon and finite at the MBO and ISCO. Their ratio varies as $\mathcal{E}_{cm} |_{r_+=M} : \mathcal{E}_{cm} |_{r_{mb}=(\frac{3+\sqrt{5}}{2})M} : \mathcal{E}_{cm} |_{r_{ISCO}=4M} = \infty : 3.23 : 2.6$. While for Schwarzschild BH the ratio of CM energy is $\mathcal{E}_{cm} |_{r_+=2M} : \mathcal{E}_{cm} |_{r_{mb}=4M} : \mathcal{E}_{cm} |_{r_{ISCO}=6M} = \sqrt{5} : \sqrt{2} : \frac{\sqrt{13}}{3}$. We further showed that this ratio for GMGHS BH varies as $\mathcal{E}_{cm} |_{r_+=2M} : \mathcal{E}_{cm} |_{r_{mb}=2M} : \mathcal{E}_{cm} |_{r_{ISCO}=2M} = \infty : \infty : \infty$.

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